Master 2 internship (2027)

Analysis and optimal control of a plant-bacteria mathematical model

This 6-month Master's internship will take place at the Inria Centre of Université Côte d'Azur, located in Valbonne (Sophia Antipolis, near Nice). The position is paid according to the standard Inria internship hourly rate. The intern will also benefit from additional advantages, including reduced-price access to the Inria cafeteria and partial reimbursement of transportation costs.

Place of internship: Inria Sophia Antipolis (Nice area)
Monthly allowance: approximately €660

Centre Inria d'Université Côte d'Azur, 2004 Rte des Lucioles, 06902, Valbonne.

The internship will be conducted within the **MACBES** team at Inria Sophia Antipolis with Valentina Baldazzi (INRAE) and Jean-Luc Gouzé (Inria), and within the **GREENOWL** team at Inria Sophia Antipolis with Walid Djema (Inria). The M2 student will also have the opportunity to interact with colleagues from the Sophia Agrobiotech Institut (INRAE, Université Côte d'Azur), which is geographically close, and in particular with members of IRL team, expert in plant-rhizobia symbiosis.

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Description of the Master's research topic

Soil microbial communities provide essential ecosystem services to plants, including indirect pathogen protection, enhanced stress tolerance and improved nutrient uptake [1]. Recent evidence indicates that plant carbon allocation to the microbial compartment— hereafter referred to as PCAM — plays a pivotal role in shaping the composition and activity of associated microbial communities. Plant-derived carbon can act as signaling molecules, carbon substrates, or exert antibiosis against specific microbial groups. Under biotic and abiotic stress, plants may substantially modify their PCAM profiles, by preferentially allocate carbon to beneficial, high-quality partners [2] . Such phenomena can be investigated through mathematical modeling.

The objective of this internship is to formulate and analyze a minimal mathematical model of plant–microbial interactions, explicitly incorporating the effect of PCAM on microbial growth. Interactions between plants and N-fixing bacteria will be used as a case study. Several model variants, based on systems of ordinary differential equations, may be developed. The first phase will focus on dynamical systems analysis, examining equilibria, stability and potential bifurcations, and may include model reduction using time-scale separation.

In a second phase, PCAM will be treated as a control variable. Classical control and optimal control methodologies will be applied, including Pontryagin's Maximum Principle (see, *e.g.,* [3]) and direct numerical approaches such as BOCOP and OptimalControl.jl ([4]), in order to investigate carbon allocation strategies that optimize plant growth under contrasting environmental conditions and competing bacterial strains.

Some references:

[1] Bulgarelli, D., Schlaeppi, K., Spaepen, S., van Themaat, E. V. L. & Schulze-Lefert, P. *Structure and Functions of the Bacterial Microbiota of Plants*. Annu. Rev. Plant Biol. 64, 807–838 (2013).

[2] Boivin & Lepetit, "Partner preference in the legume-rhizobia symbiosis and impact on legume inoculation strategies", Advances in Botanical Research, 94, 312-348, 2020

[3] Trélat, E. (2005). *Contrôle optimal: théorie & applications* (Vol. 36). Paris: Vuibert.

[4] Djema, W., Bayen, T. and Gouzé, J.L., 2025. *Optimal Separation of Two Microbial Species Competing for Two Substitutable Resources: Species Selection in Minimum-Time.* Journal of Optimization Theory and Applications, 205(3), p.61.

Required profile

The candidate we are looking for is a Master's student in Applied Mathematics or a related discipline, with a strong interest in biological modeling and ecological applications. A solid background in dynamical systems and solid skills in ordinary differential equations are essential. Knowledge of optimization and optimal control theory is highly recommended, and prior exposure to numerical methods for ODEs, dynamical analysis or control will be considered an asset.